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Sentiment and Speculation in a Market with Heterogenous Beliefs

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Adam Smith Workshop 2021

Models with heterogeneous beliefs have been studied extensively

- volatility in prices
- risk premia
- volume of trading (with caveats)
- real effects

What else can we learn about these endowment economies?

THIS PAPER

The model in this paper is built for tractability.

- while many aspects of the underlying mechanism are familiar, the paper is able to provide sharp characterizations

Key state variable is the wealth distribution across continuous types

- analytical characterization critical, it would be difficult to handle numerically with aggregate shocks
- continuous type distribution helps identify specific agents of interest
  - e.g., 'Mr. Market'; in a two-agent economy, this concept still exists but it is an abstract wealth-weighted average type
- evolution of the wealth distribution should be a key moment to calibrate

Continuous types make calibration to survey data more meaningful as well

PROBLEM SETUP

Model components

- Logarithmic utility over terminal consumption (Rubinstein (1974, 1976))
  - myopia, constant wealth-consumption ratios
- Complete market on a recombining binomial tree.
  - decentralization with 2-asset sequential trade
  - natural Brownian (Kogan, Ross, Wang, Westerfield (2006)) and Poisson (Chen, Joslin, Tran (2012)) continuous-time limits
- Beta-distributed beliefs about up-state probability (continuous type)
  - wealth distribution preserves functional form
  - same if extended with Bayesian learning (Beta is the conjugate prior for Binomial distribution)

Analyze results

- discrete-time examples with extreme aggregate payoffs
- continuous-time examples with Brownian and Poisson uncertainty

A PLANNER'S PROBLEM

The planner assigns Negishi (1960) weights $\lambda(h)$ and solves

$$\max_{\{C_{h,T}\}} \int_h \lambda(h) f(h) E^h [\log C_{h,T}] \, dh \qquad \text{s.t.} \int_h f(h) C_{h,T}(m) \, dh = p_T(m) \quad \forall m$$

where $p_T(m)$ is the aggregate payoff (endowment) in state m.

- $\lambda(h)$ is chosen to replicate a desired initial wealth distribution
- $E^h[\cdot]$ subjective expectations under Binomial density $g_{h,T}(m)$

Optimality conditions

$$C_{h,T}(m) = \frac{1}{\kappa(m)} \lambda(h) g_{h,T}(m) .$$

- $\kappa(m)$ Lagrange multiplier on the resource constraint (shadow price)

Changes in inequality proportional to disagreement

$$\frac{C_{h',T}(m)}{C_{h,T}(m)} = \frac{\lambda(h')}{\lambda(h)} \frac{g_{h',T}(m)}{g_{h,T}(m)}.$$

Allocation across states depend on shadow prices and subjective beliefs only

$$\frac{C_{h,T}(m')}{C_{h,T}(m)} = \frac{\kappa(m)}{\kappa(m')} \frac{g_{h,T}(m')}{g_{h,T}(m)}$$

Recursive representation of the planner's problem

- given time-t Pareto weights $\lambda(h)$, time-$t+1$ weights are

$$\begin{array}{ll} \lambda(h)\, h & \text{after an `up' realization} \\ \lambda(h)(1-h) & \text{after a `down' realization} \end{array}$$

The paper is upfront about the fact that the model is highly stylized

- calibration may not be as innocuous as it seems
- with $T = 50$ and uniform initial belief distribution, 5% of the population believes that a 70% drop in aggregate consumption will occur with at least 7.7% probability, and if it happens, the 95%-tile of the wealth distribution will be $(0.95/0.5)^{50} = 8.7 \times 10^{13}$ times richer than the median agent

For more quantitative work, it would be useful to compute, e.g., the cross-sectional Gini coefficient and study its typical volatility over time

- it raises the question of whose wealth matters for valuation (Gomez (2020))

BELIEFS AND SPECULATION

Agents in the model speculate by choosing portfolios different from the market portfolio.

- persistent belief heterogeneity well documented empirically
- but the key is to get the right relationship between beliefs and cross-sectional dispersion in portfolios

Speculation is the outcome of the tradeoff between belief heterogeneity and risk aversion

- in the Merton (1971) model

$$\pi_h = \frac{1}{\gamma} \frac{E^h [R_t] - r_f}{\sigma^2}$$

- calibrate not only belief dispersion but also the sensitivity of the risky share to subjective expected return, here controlled by γ

SOME DETAILS

1. 'Risky bond' and 'bubbly asset' terms are somewhat misleading
    - we are pricing aggregate endowment, not the payoff of a single asset
2. Many results similar to 'price impact' results in the Brownian model of Kogan, Ross, Wang, Westerfield (2006) and 'disaster impact' in the Poisson model of Chen, Joslin, Tran (2012)
    - a closer comparison would be useful

SUMMARY

A very clearly written paper, a pleasure to read

- analytical formulas make the mechanisms transparent
- helps with thinking about what matters in heterogeneous beliefs models

The model focuses on highlighting qualitative mechanisms

- above comments are largely targeted at future quantitative work