A Theory of Rational Short-Termism with Uncertain Betas

Christian Gollier

Discussed by Jaroslav Borovička (NYU)

May 16, 2014

Aim of the paper

The model

- iid consumption growth: $\Delta c_{t+1} = \mu_g + \sigma_g w_{t+1}$
- pricing of levered consumption claims: c_t^{β}
- uncertainty about β.

The question

- Implication of β uncertainty on discounting of long-term claims
  - long-term investment projects, environmental assets, social security, ...

The answer

- For plausible parameterizations, long-term assets discounted at higher rates due to uncertainty in β.
- mean grows with βt, volatility with $\beta\sqrt{t}$, higher β always dominates for large t.

Temporal decomposition of risk

Payoffs at different horizons discounted at different rates

- zero coupon bond yield curve — risk-free payoffs
- temporal decomposition of risk

Empirical

- **equity yields** — Van Binsbergen, Hueskes, Koijen, Vrugt, and others
- some evidence in favor of decreasing risk premia

Theoretical

- **shock elasticities** — Borovička, Hendricks, Hansen, Scheinkman (2011), Hansen, Scheinkman (2012), Borovička, Hansen (2013), Borovička, Hansen, Scheinkman (2014)

Shock elasticities

Risk premia reflect

- exposure of cash flows to risk
- price of this exposure (compensation for a unit of risk)

Shock exposure elasticity

- sensitivity of expected cash flow

Shock-price elasticity

- sensitivity of expected return

Expected returns and shock elasticities

- Expected return on cash flow F_t

$$\frac{E\left[F_t\right]}{E\left[S_t F_t\right]} = \frac{E\left[E\left[F_t \mid \beta\right]\right]}{E\left[E\left[S_t F_t \mid \beta\right]\right]}$$

- Payoff β perturbation $\log H(\mathrm{r}) = \mathrm{r}W$

$$\beta\left(F_t H(\mathrm{r})\right) = \beta + \mathrm{r}W$$

- Sensitivity of expected returns

$$\overbrace{\underbrace{\frac{d}{d\mathrm{r}} \log E\left[F_t H(\mathrm{r})\right]\Big|_{\mathrm{r}=0}}_{\text{shock-exposure elasticity}} - \frac{d}{d\mathrm{r}} \log E\left[S_t F_t H(\mathrm{r})\right]\Big|_{\mathrm{r}=0}}^{\text{shock-price elasticity}}$$

Shock elasticities

Experiment: A marginal increase in the dispersion of β

- consequence for expected payoffs (shock-exposure elasticity)

$$\left(1 - \sigma_g^2 \sigma_\beta^2 t\right)^{-1} \left(\mu_g + \sigma_g^2 \mu_\beta\right) \sigma_\beta$$

  - "correlated-risk-trend effect"

- consequence for expected returns (shock-price elasticity)

$$\left(1 - \sigma_g^2 \sigma_\beta^2 t\right)^{-1} \gamma \sigma_g^2 \sigma_\beta$$

Required compensations for additional risk diverge to $+\infty$.

- valuations **rise** with more risk / higher β (for typical parametrizations)
- but expected payoffs rise even more

Discussion

Suitability of the endowment economy assumption

- If expected returns are so high (infinite), why investors don't invest?
- They must dislike this type of risk.

What are the important sources of (parameter) uncertainty?

- long-run risk
- disasters

Resolving the uncertaity in β over time

- interaction with nonseparable recursive preferences
- Collin-Dufresne, Johannes, Lochstoer (2013)

Conditional expectations

- Let $\beta = \mu_\beta + \sigma_\beta W$ where $W \sim N(0,1)$. Then

$$\exp\left(a\beta + b\beta^2 + c\right) = \exp\left(a\mu_\beta + b\mu_\beta^2 + c + AW + BW^2\right)$$

with

$$A = a\sigma_\beta + 2b\mu_\beta\sigma_\beta \qquad B = b\sigma_\beta^2$$

- Further

$$E\left[\exp\left(AW + BW^2\right) f(W)\right] = k\widetilde{E}\left[f(W)\right]$$

with

$$k = (1-2B)^{-1/2}\exp\left(\frac{1}{2}\frac{A^2}{1-2B}\right)$$

and $\widetilde{E}$ is the expectation under the distribution of W:

$$W \sim N\left((1-2B)^{-1}A, (1-2B)^{-1}\right)$$

Expected returns

- Expected returns are ($C_0 = 1$)

$$\frac{E\left[F_t\right]}{E\left[S_t F_t\right]} = \frac{E\left[E\left[C_t^{\beta} \mid \beta\right]\right]}{E\left[E\left[e^{-\delta t} C_t^{\beta-\gamma} \mid \beta\right]\right]} = \frac{E\left[\exp\left(\mu_g t\beta + \frac{1}{2}\sigma_g^2 t\beta^2\right)\right]}{\left[\exp\left(-\delta t + \mu_g t\left(\beta-\gamma\right) + \frac{1}{2}\sigma_g^2 t\left(\beta-\gamma\right)^2\right)\right]}$$

- Expected payoff

$$\begin{aligned} a_t &= \mu_g t \qquad b_t = \frac{1}{2}\sigma_g^2 t \qquad c_t = 0 \\ A_t^e &= a_t\sigma_\beta + 2b_t\mu_\beta\sigma_\beta = \left(\mu_g + \sigma_g^2\mu_\beta\right)\sigma_\beta t \\ B_t^e &= b_t\sigma_\beta^2 = \frac{1}{2}\sigma_g^2\sigma_\beta^2 t \end{aligned}$$

so we get

$$\exp\left(a_t\mu_\beta + b_t\mu_\beta^2 + c_t\right)\left(1-2B_t^e\right)^{-1/2}\exp\left(\frac{1}{2}\frac{A^2}{1-2B}\right) =$$

$$= \exp\left(\mu_g\mu_\beta t + \frac{1}{2}\sigma_g^2\mu_\beta^2 t\right)\left(1-\sigma_g^2\sigma_\beta^2 t\right)^{-1/2}\exp\left(\frac{1}{2}\frac{\left[\left(\mu_g + \sigma_g^2\mu_\beta\right)\sigma_\beta t\right]^2}{1-\sigma_g^2\sigma_\beta^2 t}\right)$$

Expected returns

- Price

$$
\begin{aligned}
a_t &= \left(\mu_g - \gamma\sigma_g^2\right) t \qquad b_t = \frac{1}{2}\sigma_g^2 t \qquad c_t = \left(-\delta - \gamma\mu_g + \frac{1}{2}\gamma^2\sigma_g^2\right) t \\
A_t^e &= a_t\sigma_\beta + 2b_t\mu_\beta\sigma_\beta = \left(\mu_g - \gamma\sigma_g^2 + \sigma_g^2\mu_\beta\right)\sigma_\beta t \\
B_t^e &= b_t\sigma_\beta^2 = \frac{1}{2}\sigma_g^2\sigma_\beta^2 t
\end{aligned}
$$

  - so we get

$$
\begin{aligned}
& \exp\left(a_t\mu_\beta + b_t\mu_\beta^2 + c_t\right)(1-2B_t^e)^{-1/2}\exp\left(\frac{1}{2}\frac{A^2}{1-2B}\right) = \\
= \quad & \exp\left(\left(\mu_g - \gamma\sigma_g^2\right)\mu_\beta t + \frac{1}{2}\sigma_g^2\mu_\beta^2 t + \left(-\delta - \gamma\mu_g + \frac{1}{2}\gamma^2\sigma_g^2\right) t\right) \cdot \\
& \cdot \left(1 - \sigma_g^2\sigma_\beta^2 t\right)^{-1/2}\exp\left(\frac{1}{2}\frac{\left[\left(\mu_g - \gamma\sigma_g^2 + \sigma_g^2\mu_\beta\right)\sigma_\beta t\right]^2}{1-\sigma_g^2\sigma_\beta^2 t}\right)
\end{aligned}
$$

- The question is what explodes faster. Expected payoff explodes faster iff

$$
\begin{aligned}
\left[\left(\mu_g + \sigma_g^2\mu_\beta\right)\sigma_\beta t\right]^2 &> \left[\left(\mu_g - \gamma\sigma_g^2 + \sigma_g^2\mu_\beta\right)\sigma_\beta t\right]^2 \\
\mu_\beta &> \frac{1}{2}\gamma - \frac{\mu_g}{\sigma_g^2}
\end{aligned}
$$

Shock elasticities

- Consider a perturbation $\log H(\mathrm{r}) = \mathrm{r}W$. Consider

$$M_t = \exp\left(A_t W + B_t W^2\right)$$

Then the shock elasticity for M is

$$\frac{d}{d\mathrm{r}} \log E\left[M_t H(\mathrm{r})\right]\Big|_{\mathrm{r}=0} = \frac{E\left[M_t W\right]}{E\left[M_t\right]} = \widetilde{E}\left[W\right] = \left(1 - 2B_t\right)^{-1} A_t$$

- Shock-exposure elasticity for C_t^β

$$\begin{aligned} a_t &= \mu_g t \qquad b = \frac{1}{2}\sigma_g^2 t \\ A_t^e &= a_t \sigma_\beta + 2 b_t \mu_\beta \sigma_\beta = \left(\mu_g + \sigma_g^2 \mu_\beta\right)\sigma_\beta t \\ B_t^e &= b_t \sigma_\beta^2 = \frac{1}{2}\sigma_g^2 \sigma_\beta^2 t \end{aligned}$$

So the shock elasticities ceases to exist for $t > \left(\sigma_g^2 \sigma_\beta^2\right)^{-1}$.

- The elasticity is

$$\left(1 - \sigma_g^2 \sigma_\beta^2 t\right)^{-1} \left(\mu_g + \sigma_g^2 \mu_\beta\right)\sigma_\beta t$$

Shock elasticities

- Shock-value elasticity for C_t^β, i.e., shock elasticity for $S_t C_t^\beta = e^{-\delta t} C_t^{\beta-\gamma}$

$$\begin{aligned} a_t &= \left(\mu_g - \gamma\sigma_g^2\right) t \qquad b = \frac{1}{2}\sigma_g^2 t \\ A_t^v &= a_t\sigma_\beta + 2b_t\mu_\beta\sigma_\beta = \left(\mu_g - \gamma\sigma_g^2 + \sigma_g^2\mu_\beta\right)\sigma_\beta t \\ B_t^v &= b_t\sigma_\beta^2 = \frac{1}{2}\sigma_g^2\sigma_\beta^2 t \end{aligned}$$

  So the shock elasticities ceases to exist for $t > \left(\sigma_g^2\sigma_\beta^2\right)^{-1}$ as well

- Shock-price elasticity

$$\begin{aligned} \left(1-2B_t^e\right)^{-1} A_t^e - \left(1-2B_t^v\right)^{-1} A_t^v &= \left(1-2B_t^e\right)^{-1}\left(A_t^e - A_t^v\right) = \\ &= \left(1-\sigma_g^2\sigma_\beta^2 t\right)^{-1}\gamma\sigma_g^2\sigma_\beta t \end{aligned}$$

- **Note**: because all the above elasticities assume an increase in exposure (β) in all periods, it is appropriate to rescale them by t.