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The Market Cost of Business Cycle Fluctuations

Discussion by Jaroslav Borovička (NYU and Federal Reserve Bank of Minneapolis)

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HOW COSTLY ARE ECONOMIC FLUCTUATIONS?

Lucas (1987): $\frac{1}{2}\gamma\sigma_c^2 \approx 0.05\%$ of aggregate consumption

- based on a CRRA preference ($\gamma \approx 1$), iid consumption growth calculation
- inconsistent with evidence on risk premia (Hansen and Singleton (1983))

Subsequent literature

- introduced more relevant sources of risk (Krebs (2007))
- used asset price data to discipline the pricing of risk (Alvarez and Jermann (2004))

This paper

- cost of all consumption fluctuations large (as in Alvarez and Jermann (2004))
- but cost of fluctuations at business cycle frequencies also sizable (unlike in Alvarez and Jermann (2004))

COMMENTS

Measuring the cost of business cycles

- welfare cost and risk premia
- discounting
- filtering the business cycle component

Stochastic discount factors and pricing

- Cressie–Read divergence family
- term structure of risk premia
- conditional valuation

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MARGINAL AND TOTAL COST OF ECONOMIC FLUCTUATIONS

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$\Omega'(0)$ is the marginal cost of economic fluctuations

$$\Omega'(0) = \frac{V'(C)\left(C^{tr} - C\right)}{V'(C)\,C} = \frac{P\left[C^{tr}\right] - P[C]}{P[C]}$$

- $P\left[C^{tr}\right]$ and $P[C]$ can be inferred from asset price data (if traded)

MARGINAL AND TOTAL COST OF ECONOMIC FLUCTUATIONS

A simple iid growth setting (risk aversion γ)

$$\log C_{t+1} - \log C_t = \mu + \sigma_c W_{t+1}$$

Marginal cost: $\Omega'(0) \approx \gamma\sigma_c^2$

- changing risk exposure $\sigma_c \to 0$, holding price of risk $\gamma\sigma_c$ constant

Total cost: $\Omega(1) \approx \frac{1}{2}\gamma\sigma_c^2$

- changing risk exposure $\sigma_c \to 0$ and price of risk $\gamma\sigma_c \to 0$

$$\int_0^{\sigma_c} \gamma\sigma d\sigma = \frac{1}{2}\gamma\sigma_c^2$$

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Under (pretty) general conditions the marginal cost is an upper bound:

$$\Omega(1) < \Omega'(0)$$

$$\Omega'(0) = \frac{P\left[C^{tr}\right]}{P[C]} - 1 = \frac{1 + E\left[R_c^{tr}\right]}{1 + E[R_c]} \frac{E[R_c] - g_c}{E\left[R_c^{tr}\right] - g_c} - 1$$

Cost of all consumption fluctuations: $C_t^{tr} = \overline{C}(1 + g_c)^t$ and $E\left[R_c^{tr}\right] = R_f$

Cost of business cycle fluctuations: C_t^{tr} with business cycle freqs removed

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- $\Omega'_{total}(0)$ very large (30% or (much) more)
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Questions (in the context of the current paper)

1. Determination of $R_f - g_c$ (risk-free rates)
2. Determination of $E[R_c]$ (consumption risk premium)
3. Determination of 'trend' consumption (filtering out business cycles)

$$\Omega'(0) = \frac{P\left[C^{tr}\right]}{P\left[C\right]} - 1 = \frac{1+R_f}{1+E\left[R_c\right]} \frac{E\left[R_c\right]-g_c}{R_f-g_c} - 1$$

Alvarez and Jermann (2004) calibrate $R_f = 2.16\%$, $g_c = 1.93\%$.

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Additional 20 years of data (Blanchard (2019))

- Do historical data still support $R_f > g_c$?
- With $R_f \approx g_c$, the calculation becomes fragile ($\Omega'(0) = \infty$ when $R_f = \sigma_c$)

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This paper: focus instead at the term structure of finite horizon risk

- characterize welfare gains from stabilizing the next $j = 1, \ldots, J$ years of consumption uncertainty
- more robust to choice of risk-free discounting

CONSUMPTION RISK PREMIUM

$$\Omega'(0) = \frac{P\left[C^{tr}\right]}{P[C]} - 1 = \frac{1+E\left[R_c^{tr}\right]}{1+E[R_c]} \frac{E[R_c]-g_c}{E\left[R_c^{tr}\right]-g_c} - 1$$

Alvarez and Jermann (2004) project consumption growth

$$\frac{C_{t+1}}{C_t} = \underbrace{\beta' R_{t+1}^{ref}}_{\text{priced by arbitrage}} + u_{t+1}$$

- A1: risk in u_{t+1} not priced $\implies P[C_{t+1}/C_t] = \beta'\mathbf{1} \implies$ consumption risk premium $E[R_c]$
- A2: pricing of risk in u_{t+1} satisfies Cochrane and Saa-Requejo (2000) good deal bounds $\implies$ upper bound on consumption risk premium

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This paper: construct an 'entropy-based' I-SDF and use it to price consumption growth $\implies$ sharper pricing results

Filtering out all consumption fluctuations

- replace consumption with a deterministic trend

$$C_t^{tr} = C_0 (1 + g_c)^t \qquad\qquad 1 + g_c = E\left[\frac{C_{t+1}}{C_t}\right]$$

Filtering out business cycle frequencies

- apply a specific linear filter to obtain C_t^{tr} time series

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Idea: Construct trend consumption C^{tr} that removes business cycle risk.

- What constitutes 'business cycle' risk in an atheoretical model?
- Hodrick and Prescott (1980, 1997), Ravn and Uhlig (2002): A smoothing procedure that filters out particular frequencies (∼up to 8 year period).
- Two-sided filter $\implies$ C_t^{tr} depends on C_{t-j} as well as C_{t+j}, $j = 0, 1, 2, \ldots$

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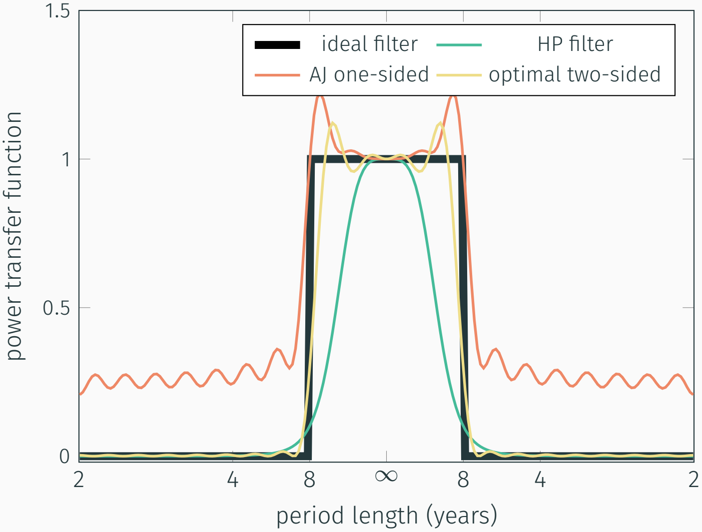
Alvarez and Jermann (2004) use a one-sided filter

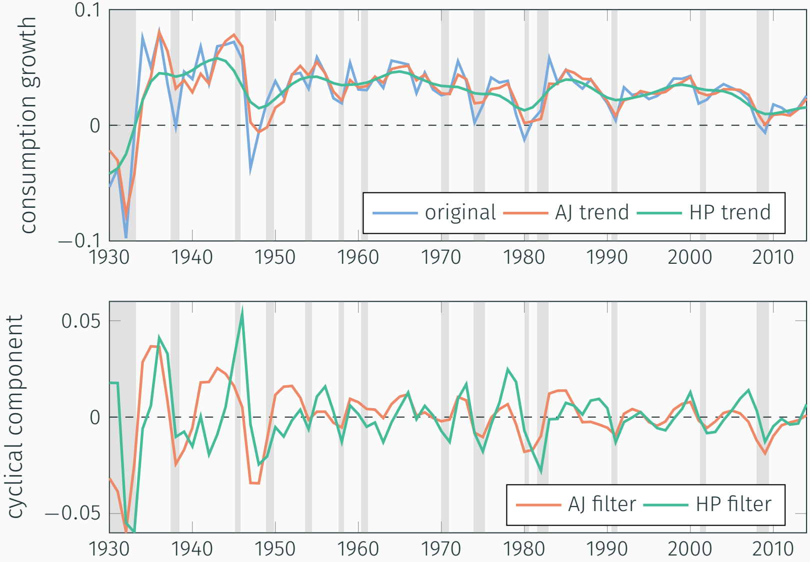
$$C_t^{tr} = a_0 C_t + a_1 \left(1 + g_c\right) C_{t-1} + \ldots + a_K \left(1 + g_c\right)^K C_{t-K}$$

- coefficients $a_0, \ldots, a_K$ chosen optimally to approximate a low pass filter

FILTERING COMPARISONS — SPECTRAL DOMAIN



FILTERING COMPARISONS — TIME DOMAIN



Assume SDF of the form

$$s_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \psi_{t+1}$$

- ψ_{t+1} represents misspecification of the CRRA model

INFORMATION SDF CONSTRUCTION

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We are interested in an SDF that

- prices a vector of excess returns $\mathbf{R}^e_{t+1}$ correctly
- has minimum dispersion of the misspecification

Problem setup

$$\min_{\psi_{t+1}} E\left[\phi\left(\psi_{t+1}\right)\right] \qquad \text{s.t. } E\left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \psi_{t+1} \mathbf{R}^e_{t+1}\right] = 0$$

THE CRESSIE–READ FAMILY OF DIVERGENCES

$$\min_{\psi_{t+1}} E\left[\phi\left(\psi_{t+1}\right)\right] \qquad \text{s.t. } E\left[s\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\psi_{t+1}\mathbf{R}^e_{t+1}\right] = 0$$

Cressie and Read (1984) family of divergences

$$\phi\left(r\right) = \frac{1}{\theta\left(1+\theta\right)}\left[r^{1+\theta} - 1\right]$$

- $\theta = 1 \implies \phi\left(r\right) = \frac{1}{2}\left(r^2 - 1\right)$: Hansen and Jagannathan (1991)
- $\theta = 0 \implies \phi\left(r\right) = r\log r$: Stutzer (1995)
- $\theta = -1 \implies \phi\left(r\right) = -\log r$: Alvarez and Jermann (2005), this paper

Convenient dual minimization problem in terms of Lagrange multipliers on the constraints

$$\widehat{\psi}_t = \frac{1}{T\left(1 + \widehat{\theta}' \mathsf{R}^e_{t+1} \left(C_{t+1}/C_t\right)^{-\gamma}\right)}$$

with

$$\widehat{\theta} = \arg\min_{\theta} - \sum_{t=0}^{T-1} \log\left(1 + \theta' \mathsf{R}^e_{t+1} \left(C_{t+1}/C_t\right)^{-\gamma}\right)$$

1. Replace consumption C_{t+j} with C^{tr}_{t+j} for $j = 1, \ldots, J$
2. Construct the term structure of (cumulative) welfare cost

$$\Omega'_{[J]}(0) = \frac{P_t\left[C^{tr}_{t+1}\right] + P_t\left[C^{tr}_{t+2}\right] + \ldots + P_t\left[C^{tr}_{t+J}\right]}{P_t\left[C_{t+1}\right] + P_t\left[C_{t+2}\right] + \ldots + P_t\left[C_{t+J}\right]} - 1$$

   as a function of J

3. Compare results
    - information SDF
    - CRRA model ($\psi_{t+1} = 1$)
    - Lucas calculation with lognormal consumption ($\gamma = 10$)

RESULTS

Table 2A: nondurable consumption, pricing market portfolio

	all fluctuations									
I-SDF	1.52	5.08	11.72	14.53	14.87	14.71	14.30	14.07	14.01	13.92
CRRA	0.94	2.02	3.43	4.40	4.68	4.82	4.87	4.92	4.99	5.03
Lucas	0.76	1.10	1.41	1.69	1.95	2.18	2.39	2.57	2.73	2.87
	business cycle fluctuations									
I-SDF	0.56	1.44	3.32	3.86	3.54	3.26	2.99	2.80	2.73	2.67
CRRA	0.47	0.82	1.20	1.37	1.32	1.26	1.21	1.19	1.18	1.17

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Table 2A: redo with one-sided filter (Alvarez and Jermann (2005) trend)

	business cycle fluctuations									
I-SDF	0.36	0.94	1.92	1.64	1.10	0.81	0.66	0.61	0.63	0.63
CRRA	0.20	0.30	0.39	0.34	0.23	0.18	0.16	0.15	0.16	0.16

SUMMARY OF THE RESULTS

Filtering of the business cycles changes results substantially

- results in line with Alvarez and Jermann (2005) good deal bounds assumption (0.84% welfare cost)
- good deal bounds are an assumption on any SDF, including I-SDF
- I-SDF still useful, sharper determination of cost, rather than bounding

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- make the determination of $E_t[s_{t+1}]$ more explicit

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Results indicate increasing term structure of welfare cost (risk premia)

- at least for horizons 1–5 years, then dropoff
- consistent with Koijen and van Binsbergen (2017) for S&P500 dividend strips

INCLUDING CONDITIONING INFORMATION

We would now like to include conditioning information

$$\min_{\psi_{t+1}} E_t\left[\phi\left(\psi_{t+1}\right)\right] \qquad \text{s.t. } E_t\left[s\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\psi_{t+1}\mathbf{R}^e_{t+1}\right] = 0$$

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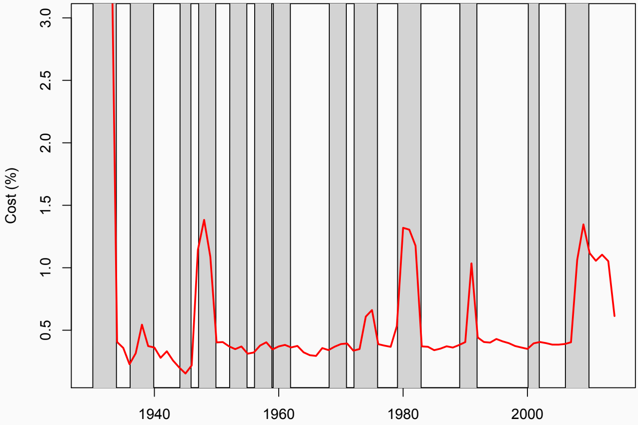
How to implement this empirically?

- conventional approach: include (linear) instruments and condition down
- this paper: use smoothed empirical likelihood approach of Kitamura, Tripathi and Ahn (2004)
    - recovers nonparameteric conditional transition density over a vector of state realizations (here MA of lagged consumption growth)
    - see Ghosh and Roussellet (2019) for details



Cost strongly countercyclical as expected

- countercyclical risk premia, especially for short-horizon strips

CONCLUSION

I-SDF construction appealing

- simple to implement, authors find very good pricing performance
- explore how results differ across the Cressie and Read (1984) class
- why misspecification relative to a CRRA model? Other options?
- does ψ_{t+1} reflect SDF misspecification or subjective beliefs?

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- predictability issues with the HP filter (especially for conditional results)
- sharpen the comparison to Alvarez and Jermann (2005)
- isolating permanent vs transitory components? (Borovička, Hansen and Scheinkman (2016))

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What about the cost of uninsurable idiosyncratic risk?

- may be still large at business cycle frequencies