Asset Pricing in the Frequency Domain: Theory and Empirics

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Discussed by Jaroslav Borovička

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Framework

- sources of (macro)economic risk

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- preferences as an aggregator of the risk sources (innnovations representation)

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- innovation to the SDF

$$\Delta E_{t+1}[m_{t+1}] = -\left(\sum_{k=0}^{\infty} z_k g_k\right) \varepsilon_{t+1}$$

  - correlation between $\{z_k\}$ and $\{g_k\}$

Discrete-time Fourier transform

- representation in the frequency domain

$$G(\omega) = \sum_{k=0}^{\infty} g_k e^{-i\omega k} \qquad Z(\omega) = \sum_{k=0}^{\infty} z_k e^{-i\omega k}$$

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$$\sum_{k=0}^{\infty} z_k g_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} Z(\omega) G(\omega)\, d\omega$$

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This paper: What can we learn from the spectral decomposition of preferences $Z(\omega)$

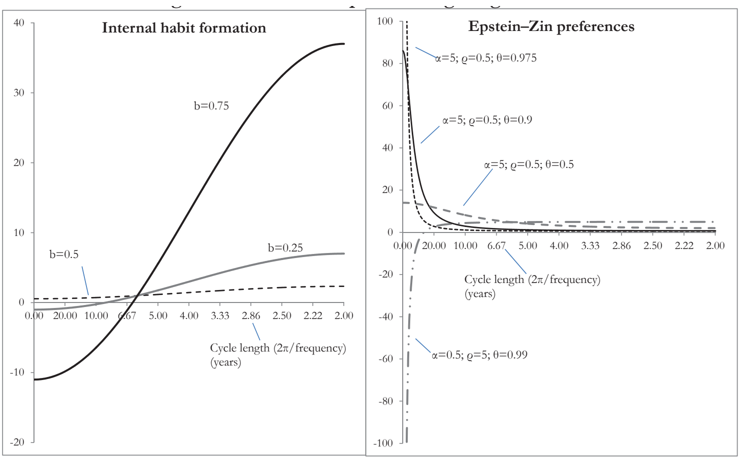
- Estimate $\{g_k\}$ $(G(\omega))$ from data (VAR)
- Estimate different specifications for $Z(\omega)$
  - Some are linearizations of conventional preferences
  - Others have more statistical basis: aversion to risk at different frequencies

Goals

Why do we do all this?

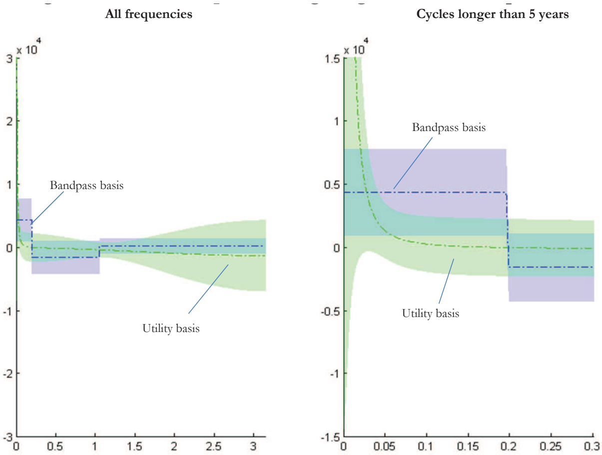
1. Intuition
   - How do preferences load on different frequencies?
2. Estimation
   - Spectral decomposition cannot bring in any new information.
   - What if models are misspecified?
   - Estimating reduced form preference specification in the frequency domain.

Intuition



- Aversion to / preference for persistence

Estimation


All frequencies
Cycles longer than 5 years
x 10^4
Bandpass basis
Utility basis
3
2
1
0
-1
-2
-3
0
0.5
1
1.5
2
2.5
3
1.5
0.5
-0.5
-1.5
0.05
0.1
0.15
0.2
0.25
0.3

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- Bandpass filter seems like an interesting reduced-form idea.
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  - Compare with tranching of mortgage-backed securities.
  - What if we take the model seriously and start fishing for cash flows which are underpriced/overpriced?
  - Very similar cash flows with frequencies concentrated around the steps should be priced quite differently.
  - **Security design**: spuriously attractive investment opportunities with very high Sharpe ratios.

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- Multiple periods
  - **this paper**: impact of low-frequency components on the one-period SDF

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- Approximation errors
  - logarithms vs levels
  - loglinear approximation $\Longrightarrow$ bandpass filter $\Longrightarrow$ what happens to the SDF in levels?