Anne G. Balter, Pascal J. Maenhout, and Hao Xing

Model Ambiguity versus Model Misspecification in Dynamic Portfolio Choice and Asset Pricing

Discussion by Jaroslav Borovička (NYU)

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Partly based on work by Kenji Wada (NYU).

1. Structured and unstructured models and agents' attitudes to uncertainty.
2. Portfolio choice under model ambiguity and model misspecification.
3. How to model interesting structured models?
    - implementing cross-equation restrictions

How should agents cope with lack of knowledge of the correct model?

Agent may have in mind a set of (parameterized) theories.

- agent may not be sure which of the theories is correct: model ambiguity
- these theories constitute so-called structured models

Agent may also be concerned that each of the theories is only an approximation.

- deviations from the theories have unstructured form
- agent is thus coping with model misspecification

See Hansen and Sargent (2021, 2022).

Representation of ambiguity/misspecification concerns

- entropy measures statistical distance between alternative models (probability distributions)

Admissibility

- admissible decision rules cannot be dominated by others across all models

Dynamic consistency

- concerns and decisions expressed today are consistent with future choices

These three aspects impose critical restrictions on the modeling of sets of structured and unstructured models.

Max-min decision rules

A variety of approaches in this literature shares the max-min approach.

- decision-maker devises decision rules that maximize utility under a given model
- malevolent ‘nature’ chooses models that minimize utility given the decision rules

The resulting saddle point is the outcome of this two-person game.

- decision rules that are dynamically consistent under each of the models may not be dynamically consistent in the max-min problem
- attempts to rectify dynamic consistency by adjusting the set of models (see rectangularity, Epstein and Schneider (2003)) may clash with admissibility.

This papers revisits the Merton (1971) portfolio choice problem.

The decision problem is now enriched by including ambiguity and model misspecification concerns.

- a key contribution is the modeling of the sets of structured and unstructured models
- these are completely novel aspects both from the theoretical and applied perspective
- while doing so, the authors have to cope with the conceptual issues outlined previously

Modeling sets of models

A 'benchmark' iid model for returns on the risky asset

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB_t^{\mathbb{B}}. \tag{1}$$

The agent solves

$$\max_{c,\pi} \min_{U \in \Theta} \mathbb{E}^U \left[\int_0^\infty \delta e^{-\delta t} U(c_t)\, dt \right]$$

subject to the resource constraint.

The set Θ includes (a subset of) models of returns that are 'close' to (1).

- statistical distance is expressed using initial entropy budget E_0

$$E_0 = \mathbb{E}_0^U \left[\int_0^\infty \delta e^{-\delta t} \frac{1}{2} u_t^2 dt \right]$$

- under the distorted model, risky returns follow

$$\frac{dS_t}{S_t} = (\mu - \sigma u_t)\, dt + \sigma dB_t^U$$

Entropy evolution

Alternative models impose different time- and state-dependent distortions.

- this is achieved by 'spending' the entropy budget in specific ways across time and space

The martingale representation theorem implies the representation

$$dE_t = \delta E_t dt - \frac{\delta}{2} u_t^2 dt + g_t dB_t^U$$

- u_t controls current distortion, which reduces future entropy budget
  - more distortion today implies less distortion in the future
- g_t allocates entropy budget across alternative next-period states
  - $g_t > 0$ ($g_t < 0$) allocates distortions procyclically (countercyclically)

How is entropy evolution linked to structured and unstructured model sets?

The authors make the following modeling choices.

Unstructured models

- a full set of models restricted only by the initial entropy budget E_0
- this is equivalent to ‘nature’ having full control over u_t and g_t

Structured models

- models in which entropy budget cannot be allocated in a state-dependent fashion
- malevolent ‘nature’ controls only u_t, while $g_t = 0$

Why are those choices interesting?

- in this setup, they have a particular interpretation and results

Unstructured models

- state-dependent entropy allocation interacts with CRRA utility
- depending on risk aversion γ, emphasizes hedging or speculative motives associated with subjective persistence of returns

Structured models

- iid benchmark model with state-independent entropy allocation implies that structured models again have iid returns (with different mean)

Results

When $\gamma > 1$, a rational agent would like to use the risky asset for hedging purposes when returns are mean-reverting.

- malevolent ‘nature’ therefore chooses a worst-case model with persistent returns

How? Assign more entropy into states following negative returns.

- therefore $g_t < 0$
- agent believes returns are persistent, chooses negative hedging demand

The paper provides an extensive characterization of a range of results.

- belief characterization, dynamic evolution, equilibrium, ...

Recursive representation requires keeping track of the entropy variable.

- an auxiliary state variable representing commitment

Some of the results are qualitatively similar to Bayesian learning

- experience effects, vanishing sensitivity over life cycle
- sharply distinguishing Bayesian learning consequences of model misspecification would be highly desirable

Modeling the sets of structured and unstructured models is novel and interesting.

- recall the vision that structured models represent interesting economic theories
- not clear how optimization only over u_t leads to such a set in general environments
- Hansen and Sargent (2022) proceed differently but the same concern applies to their analysis

This part is based on work and results by Kenji Wada (NYU, in progress).

Let us take more seriously the idea that structured models constitute 'interesting' economic theories that the agent is uncertain about.

Wada (2023) extends the financial accelerator model of He and Krishnamurthy (2012, 2019) to include such structured uncertainty.

- risky asset is a claim on aggregate dividend with drift μ and volatility σ
- financially constrained specialists form levered portfolios
- an adverse shock tightens the constraint because specialists lose wealth share
- compensation for leverage requires higher expected returns and generates state-dependent return volatility

Specialists' decision problem critically depends on observable information and their ability to infer structural relationships.

- specialists observe endogenous returns dR_t, and can infer $\sigma_{R,t}$
- they understand the cross-equation restriction that relates $\sigma_{R,t}$ to structural parameters of the model
- they are however uncertain about those parameters
- many different parameter combinations lead to the same $\sigma_{R,t}$ but have different implications for the expected return $\mu_{R,t}$

This ambiguity alters portfolio choice in a max-min decision problem.

A simple example

Ambiguity about dividend volatility σ and financial constraint parameter m.

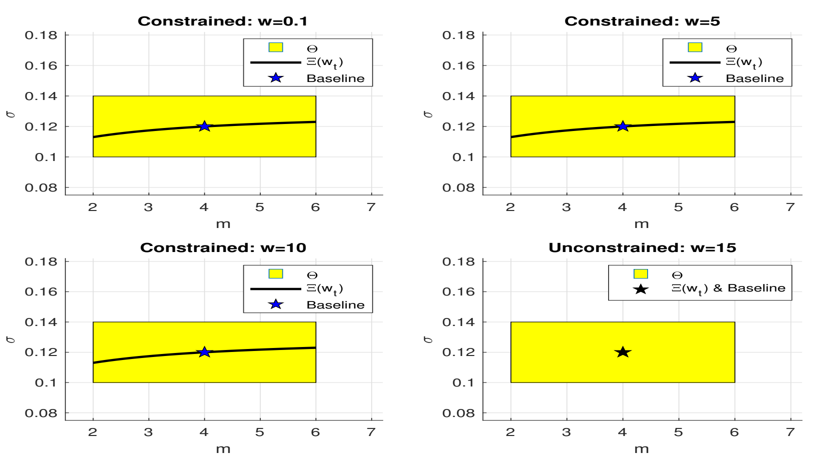
Financial constraint does not bind.

- observed return volatility: $\sigma_{R,t} = \sigma$
- specialists infer from the cross-equation restriction that the economy is unconstrained
- no ambiguity about expected return

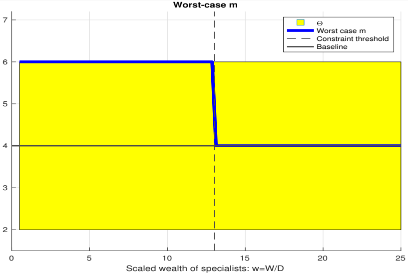
Financial constraint binds.

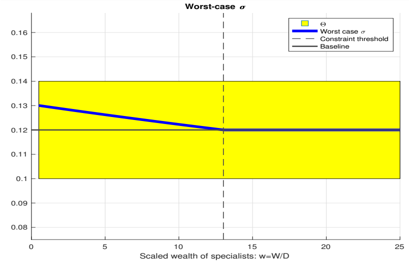
- observed return volatility: $\sigma_{R,t} = \sigma_R(m, \sigma)$
- structured models: pairs (m, σ) that satisfy the cross-equation restriction
- find structured model that has the most adverse consequence for the individual specialist

Sets constrained by cross-equation restrictions



Worst-case models





Specialists fear that financial constraints of other specialists are not tight.

- from this, they infer that expected return on risky asset is low
- to rationalize $\sigma_{R,t}$ they conclude that fundamental volatility σ must have increased
- specialists reduce demand for risky asset

In equilibrium, this must further reduce equilibrium prices

- ambiguity is state-dependent, and amplifies risk premia in the tail

Balter, Maenhout, and Xing (2023) make substantial progress (!!!) in modeling structured and unstructured uncertainty.

Theoretical contributions

- using continuation entropy as a state to implement ambiguity dynamics
- structured and unstructured sets modeled by alternative assumptions on how entropy can be controlled

Applied contributions

- ambiguity under CRRA utility
- sharp implications for worst-case models that disallow hedging/speculation

Further steps

- how to model interesting structured models in more general environments $\Longrightarrow$ this paper provides rigorous conceptual foundations

Appendix

Balter, Anne G., Pascal J. Maenhout, and Hao Xing. 2023. "Model Ambiguity versus Model Misspecification in Dynamic Portfolio Choice and Asset Pricing."

Epstein, Larry G. and Martin Schneider. 2003. "Recursive Mutiple-Priors." *Journal of Economic Theory* 113 (1):1–31.

Hansen, Lars Peter and Thomas J. Sargent. 2021. "Macroeconomic Uncertainty Prices When Beliefs Are Tenuous." *Journal of Econometrics* 223 (1):222–250.

———. 2022. "Structured Ambiguity and Model Misspecification." *Journal of Economic Theory* 199:105165.

He, Zhiguo and Arvind Krishnamurthy. 2012. "A Model of Capital and Crises." *Review of Economic Studies* 79 (2):735–777.

———. 2019. "A Macroeconomic Framework for Quantifying Systemic Risk." *American Economic Journal: Macroeconomics* 11 (4):1–37.

Merton, Robert C. 1971. "Optimum Consumption and Portfolio Rules in a Continuous-Time Model." *Journal of Economic Theory* 3 (4):373–413.

Wada, Kenji. 2023. "A Macro-Finance Model with Structured Uncertainty." Job market paper, NYU.